Tax and the city

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The economic determinants of city structure

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- craftsmen learn from other craftsmen

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- pollution

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Large literature in urban economics:

- monocentric city model (Fujita, 1989, Fujita and Thisse, 2002): production concentrated at one point
- new economic geography (Fujita, Krugman, Venables 1999): competition between cities

Try to adress the problem of city structure arising endogeneously

- through market forces (Lucas and Rossi-Hansberg, 2002, Carlier-Ekeland, 2007, the first paper to use optimal transportation in that context)
- by a benevolent planner, who will seek an optimum and correct the externalities by appropriate taxes

The two processes will yield *different* structures

The market

The market for land

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- there is a single good in the economy, which all firms produce and all residents consume
- rents and surplus production leave the city

Equilibrium conditions: residents

All inhabitants are identical. Their utility is $U(c,S)$, where c is consumption and S the area they rent. U satisfies the usual conditions. All people who live at x pay the same rent $Q(x)$ and have the same take-home pay $\psi(x)$ (salary net of transportation costs), so they all solve the same problem:

$$\max_{c,S}\{U(c,S) \mid c + Q(x)S \leq \psi\}$$

The optimal value $\bar{u}$ must be the same for all x:

$$\bar{u} := \max_{c,S}\{U(c,S) \mid c + Q(x)S \leq \psi(x)\}$$

$\bar{u}$ is given exogeneously (income earned outside the city). This determines $N(\psi)$, *relative* density of residents, and $Q(\psi)$

Equilibrium conditions: firms

All firms are identical and have constant returns to scale.. The production per unit of land at y is $f(z, n(y))$, where $n(y)$ is the relative density of jobs. The *productivity* $z = z(y)$ is given by:

$$z(y) = g\left(\int \rho(y', y)\nu(y')dy'\right)$$

where $\rho \geq 0$, g is increasing and $\nu(y)$ is the *absolute* density of jobs. All firms located at y pay the same rent $q(y)$ and the same salary $\psi(y)$, and make the same profit $f(z, n) - \psi n - q(y)$, which is zero because of perfect competition:

$$0 = \max_{n \geq 0} \{f(z, n) - \psi(y) n - q(y)\}$$

This gives $n(\psi)$, relative density of jobs

Transportation costs

Workers who live at x and work at y, where they pick up a salary $\psi(y)$, have a revenue

$$\varphi(x) = \psi(y) - c(x, y)$$

with $c \geq 0$ and $c(x, x) = 0$. It may be convex or concave - the first case is typical of the structure of cities, the second of interregional trade.
A *transport plan* is a map $x \longrightarrow P_x$, where P_x is a probability on Ω. We understand $P_x(y)$ as the proportion of the residents of x who work at y.
This becomes a transport map $\tau : \Omega \longrightarrow \Omega$ if $P_x = \delta_{\tau(x)}$ is a Dirac mass, ie if all those who live at x work at the same place $\tau(x)$. This is the case when the cost is convex.

Equilibrum: definition

- We start from two densities $\mu(x)$ (residents) et $\nu(y)$ (jobs) and two functions $\varphi(x)$ (take-home pay at x) et $\psi(y)$ (salary at y) such that:

$$
\begin{aligned}
\int \mu &= \int \nu \\
\varphi(x) &= \max_y \{\psi(y) - c(x,y)\} \\
\psi(y) &= \min_x \{\varphi(x) + c(x,y)\}
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- We deduce the productivity $z(y)$, and the relative densities $N = N(\varphi(x))$ and $\nu = \nu(z(y), \psi(y))$ as well as the rents $Q = Q(\varphi(x))$ and $q = q(z(y), \psi(y))$ for residences and firms

Equilibrium: definition

- In view of $q(y)$ and $Q(x)$, the landlords allot the land. If $\theta(x)$ is the proportion of land for industrial use, we must have:

$$\theta = \begin{cases} 0 & \text{si } q(z,\psi) - Q(\varphi) < 0 \\ 0 \leq \theta(x) \leq 1 & \text{si } q(z,\psi) - Q(\varphi) = 0 \\ 1 & \text{si } q(z,\psi) - Q(\varphi) > 0 \end{cases}$$

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- We deduce new distributions $\tilde{\mu}$ and $\tilde{\nu}$ by:

$$\begin{aligned} \tilde{\mu}(x) &= (1 - \theta(z,\psi,\varphi)) N(\varphi(x)) \\ \tilde{\nu}(y) &= \theta(z,\psi,\varphi) n(z,\psi) \end{aligned}$$

and there is a unique pair $(\tilde{\varphi}, \tilde{\psi})$ such that $\int \tilde{\mu} = \int \tilde{\nu}$

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- We have an equilibrium if $(\mu, \nu, \varphi, \psi) = (\tilde{\mu}, \tilde{\nu}, \tilde{\varphi}, \tilde{\psi})$

Existence

Theorem

There is always an equilibrium

It can be checked that people move: there is no equilibrium with all residents living where they work.

R. E. Lucas, Jr. and E. Rossi-Hansberg, *On the Internal Structure of Cities*, Econometrica, vol. 70 (2002), pp. 1445-1476. (*Treats the case of a circular city with radial transport and iceberg costs*)

G. Carlier and I. Ekeland, "Equilibrium structure of an bidimensional assymmetric city". *Nonlinear Analysis TWA*, vol 8 (2007): 725 - 748.

The planner :
one sector

The planner's problem

The planner has a large empty space at her disposal, and assigns locations to firms and workers in order to maximize the social optimum

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$$w(x) \text{ density of residence at } x$$

$$f(x) \text{ density of jobs at } x$$

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- It will be endogeneously determined, as the support of w and f

The agents

- **Workers** living at x and working at y have utility:

$$
\begin{aligned}
&-t\left\|x-y\right\|^2 \\
&+\frac{\theta}{2}\int\left\|x-x'\right\|^2 w\left(x'\right)dx'+\rho\int\left\|x-y\right\|^2 f\left(y\right)dy+ \\
&+s\left(T\left(x\right)\right)-p_w\left(x\right)
\end{aligned}
$$

  where $\theta > 0$ is a positive externality and either $\rho > 0$ (convenience) or $\rho < 0$ (pollution). Transportation from x to y costs $\left\|x-y\right\|^2$ in utility. The last term is the revenue at x: salary paid at $T\left(x\right)$ minus rent paid at x

- **Firms** located at y make a profit:

$$
\frac{k}{2}\int\left\|y-y'\right\|^2 f\left(y'\right)-s\left(y\right)-p_f\left(y\right)
$$

  where the first term takes into account the externality of production

The planner's problem

It consists of minimizing

$$\frac{k}{2}\int\int\left\|y-y'\right\|^2 f\left(y\right) f\left(y'\right)+\frac{t}{2}\int\left\|x-T\left(x\right)\right\|^2 w\left(x\right)$$
$$-\frac{\theta}{2}\int\int\left\|x-x'\right\|^2 w\left(x\right) w\left(x'\right)-\frac{\rho}{2}\int\int\left\|x-y\right\|^2 w\left(x\right) f\left(y\right)$$
$$\frac{\gamma_w}{2}\int w\left(x\right)^2+\frac{\gamma_f}{2}\int f\left(y\right)^2$$

over all probability densities (w, f) and all maps T which transport w to f. The last two terms are *infrastructure costs*, which are borne by the city. They prevent everyone living and working together in one huge skyscraper This problem is far from being convex. However, existence obtains provided centripetal forces dominate centrifugal ones:

$$\rho > -\max\left\{k, \theta, \frac{t}{2}\right\}$$

The optimal shape

In the case of quadratic cost, there is an explicit solution:

- monocentric, circular, radial symmetry (this is no longer the case when transportation costs are superquadratic and infrastructure costs are small). Note the symmetry will break for $\frac{1}{p}\|x - T(x)\|^p$, $p > 2$
- bell-shaped distribution of population and jobs:

$$\begin{aligned} w(x) &= \frac{2}{\pi C_w^4}\left[C_w^2 - x^2\right]_+, \\ f(y) &= \frac{2}{\pi C_f^4}\left[C_f^2 - y^2\right]_+ \end{aligned}$$

$$\begin{aligned} (k + \rho + t)\, C_f^4 - t C_w C_f^3 &= \frac{2}{\pi}\gamma_f \\ (\theta + \rho + t)\, C_w^4 - t C_f C_w^3 &= \frac{2}{\pi}\gamma_w \end{aligned}$$

Transportation

The transportation map is linear:

$$y = Tx$$

where $T = \frac{C_f}{C_w}$ can be computed directly by:

$$\gamma_w \left[(k + \rho + t) \, T^4 - tT^3 \right] + \gamma_f \left[tT^3 - (\theta + \rho + t) \, T^4 \right] = 0$$

Leading to two cases:

- $T > 1 \iff \gamma_w \, (k + \rho) > \gamma_f \, (\theta + \rho)$: suburbs are business
- $T < 1 \iff \gamma_w \, (k + \rho) < \gamma_f \, (\theta + \rho)$: suburbs are residential

Implementing the optimum

The city levies residential taxes $R(x)$ and business taxes $B(y)$. It wants to achieve the distribution (w, f)

Developers then build residential and industrial premises and rent them at prices $p_w(x)$ and $p_f(y)$ given by:

$$\max_{p_w} \left\{ [p_w(x) - R(x)]\, w(x) - \frac{\gamma_w}{2} \int w^2 dx \right\}$$

$$\max_{p_f} \left\{ [p_f(x) - B(x)]\, f(x) - \frac{\gamma_f}{2} \int f^2 dx \right\}$$

All workers at x maximise their utility by working at Tx. They all have the same utility and all firms make zero profit. We find:

$$R(x) = R(0) + \left(\rho + \frac{k}{2}\right) x^2,$$

$$B(y) = B(0) + \left(\frac{\theta}{2}\right) y^2$$

The planner:
two sectors

The two-sector city

There are two types of jobs (amenity, and industry). Ex ante, there is one type of workers. Ex post, they will split into the two types of jobs. :

$$\int f_1(y)\, dy = \int f_2(y)\, dy = 1 = \int w_1(x)\, dx = \int w_2(x)\, dx$$

The two-sector city

$$C_1 \quad : \quad = \frac{\gamma_w}{2} \int (w_1 + w_2)^2 \text{ (congestion)}$$

$$C_2 \quad : \quad = t \int \|x - T_1(x)\|^2 w_1(x) + t \int \|x - T_2(x)\|^2 w_2(x) \text{ (transportati}$$

$$C_3 \quad : \quad = \sum_{i=1}^{2} \frac{k_i}{2} \int \int |x - y|^2 f_i(x) f_i(y) \text{ (worker/worker externalities)}$$

$$C_4 \quad : \quad = \frac{\theta}{4} \int \int |x - y|^2 (w_1(x) + w_2(x))(w_1(y) + w_2(y)) \text{(firms/firms)}$$

$$C_5 \quad : \quad = \sum_{i=1}^{2} \frac{\rho_i}{2} \int \int |x - y|^2 f_i(x)(w_1(y) + w_2(y)) \text{ (firms/workers)}$$

We assume $\rho_1 > 0$ (services) and $\rho_2 + k < 0$ (industry)

Structure of the city

Existence obtains under similar conditions (centripetal parameters should dominate centrifugal ones). We find that there are four centres to the city:

$$\int f_1(y)\, ydy, \quad \int f_1(y)\, ydy, \quad \int w_1(x)\, xdx, \quad \int w_2(y)\, ydy$$

The transportation maps T_1 and T_2 are affine, and the densities are parabolic. Surprisingly, there is segregation for business and for workers

Theorem

Assume that $k_1 + \rho_1 \neq k_2 + \rho_2$. *Then* $f_1 f_2 = w_1 w_2 = 0$

We get an *asymmetric* city, with two main districts:

- the white-collar district, surrounded by a residential suburb
- the blue-collar district, surrounded by an industrial suburb

Implementation

Residential tax and business taxes depend on the distance from thecenters:

$$R(x) = C_t + \frac{\theta}{2}(\mu - x)^2$$
$$B_i(x) = C_t + \frac{\rho_i}{2}(\mu - x)^2 + \frac{k_i}{2}(\nu_i - y)^2$$

Extensions

- Other interesting shapes when there are geographical constraints (lake- or seashore, river and bridges) and more types of jobs and workers ?

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- Other interesting shapes when there are geographical constraints (lake- or seashore, river and bridges) and more types of jobs and workers ?
- Other examples of two- or more-sided platforms where interaction between users can be controlled ?
- Use concave costs, like $\|x - T(x)\|^{\alpha}$ with $\alpha < 2$ to study the effect of long-range hauls in economic geography