Job Mobility of Couples: Shapley-Scarf Markets with Coalitional Trade

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A Quick overview

- Growing literature on the engineering of school choice (2-sided matching) with several theoretical studies of situations involving externalities:

- Dutta, Masso (1997) Sasaki, Toda (1996) Roth, Peranson (1999) Pycia, Yenmez (2017) Klaus and Klijn (2005) Kojima, Pathak, Roth (2013) Ashlagi, Braverman, Hassidim (2014) Echenique, Yenmez (2007) Pycia (2012) Inal (2015) Ashlagi, Shi (2014) Dur, Wiseman (2015)

$\longrightarrow$ *externalities among couples, peer effects, student assignment problem with neighbors*

- But very little about trade, or re-assignment: job mobility

$\longrightarrow$ *why? The convincing top trading cycle algorithm (Gale): implements a competitive equilibrium, Core stable, Pareto if no indifference, and strategy-proof*

- *The TTC algorithm:* 5 jobs, 5 individuals, individual i assigned to job i

$$\left(\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ \hline 2 & 4 & 1 & 1 & 4 \\ 3 & 5 & 4 & 3 & 2 \\ 4 & 3 & 5 & 5 & 1 \\ \ldots & \ldots & \ldots & \ldots & 3 \end{array}\right)$$

Row 1 cycle: $1 \longrightarrow 2 \longrightarrow 4 \longrightarrow 1$: remove 1,2,4 from the picture

$$\left(\begin{array}{ccccc} 1out & 2out & 3 & 4out & 5 \\ \hline . & & . & . & . \\ . & & . & . & . \\ . & & 5 & . & . \\ \ldots & \ldots & \ldots & \ldots & 3 \end{array}\right)$$

Final cycle: $3 \longrightarrow 5 \longrightarrow 3$

$\sigma = (2, 4, 5, 1, 3)$ competitive allocation, Pareto, and Core stable

- *Shapley-Scarf market (Shapley, Scarf 1974)*
  - n indivisible goods (set J), n individuals (set I)
  - allocation $\sigma =$ bijection $I \to J$
  - $\sigma^0 =$ initial allocation
  - individual $i \to$ ranking of goods $\succ_i$

- *Interpretation:*
  - goods = jobs
  - trade = job mobility scheme

- *Our goal:*
  - investigating properties of centralized job mobility procedures **involving couples**
  - centralized: individual submit preferences over jobs
  - externalities among partners $\to$ what happens to my partner matters!
  - **2 types of externality: distance (please let us be close to each other) and love (please my partner as well as myself)**

JOB MOBILITY OF COUPLES: WHEN DISTANCE MATTERS

- Fixed marriage structure: partition of I into couples and singles

- Fixed geographical structure: partition of jobs into locations

- *Individuals* submit preferences over jobs

- Individuals compare allocations according to the quality of their own job and the distance to partner: comparing preferences does not resume to comparing jobs

Preferences over allocations

- $i \longrightarrow$ linear order $\succ_i$ over $\{1,...,n\} \times \{0,1\}$
  discrete metric
  $(k,0) =$ job with rank k at distance 0 to partner,
  $(k,1) =$ job with rank k at distance 1 to partner

- **Responsiveness**:
  $k < k' \Rightarrow (k,x) \succ_i (k',x)$ for all x
  $(k,0) \succ_i (k,1)$ for all k

- Trade-off between job quality and distance
  rank $k \longrightarrow$ switch $\phi(k) = \max\{1 \leq t \leq n : (t,0) \succ_i (k,1)\}$

- 2 polar cases:
  - **Job oriented** preferences: $\phi(k) = k$ for all k
  - **Distance oriented** preferences: $\phi(k) = n$ for all k

Question: can one design a centralized procedure with Core stable outcome?

- CORE C: allocation σ is Core stable if the initial allocation is not unanimously preferred by any couple to σ (weak couple rationality), and there no coalition S can be formed such that
  - it does not break couples: $C = \{i, i'\}$, $i \in S \Rightarrow i' \in S$
  - all S members are strictly better off by realloca-tiong jobs within S

- STRONG CORE C^S: Same with couple rationality replaced by standard individual rationality

Example

$$\left(\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 2 & 2 & 4 & 4 & 6 & 5 \\ 6 & 4 & 6 & 2 & 1 & 4 \\ 1 & 3 & 3 & 5 & 3 & 2 \\ 3 & 5 & 1 & 3 & 5 & 6 \\ \dots & \dots & \dots & \dots & \dots & \dots \end{array}\right)$$

couples $\{1,2\},\{3,4\},\{5,6\}$ Locations $L_1 = \{1,3,5\}$, $L_2 = \{2,4,6\}$

$\sigma = (1,3,2,4,5,6)$, $\sigma' = (6,2,3,4,1,5)$

$S = \{1,2,5,6\}$

$$\left(\begin{array}{c|cccc} & 1 & 2 & 5 & 6 \\ \hline \sigma & (3,1) & (3,1) & (4,1) & (4,1) \\ \sigma' & (2,0) & (1,0) & (2,0) & (1,0) \end{array}\right)$$

$\rightarrow$ σ is not (strong) Core stable

MAIN RESULTS

1 The Core may be empty under responsive preferences

2 The strong Core is non empty under job oriented preferences: standard TTC works!

3 Distance oriented preferences without singles: the geography matters
- If all couples can be made close (0-solvability), the Core is always non-empty, while the strong Core may be empty. based on an algorithm that fails strategy-proofness
- If at least one couple (or more) must stay distant, the Core may be empty

4 Distance-oriented preferences with singles: the disturbing latin lover
- Even under 0-solvability, the Core may be empty

JOB MOBILITY OF COUPLES: THE POINT SYSTEM

- Point system rather widespread in real-life procedures:
  - individuals are entitled to get as new position only those for which they accumulated enough points
  - point results from history of career

- Similar in spirit to the price mechanism
  - (much) simplifying assumption: value in points of current job = 'income', and 'price' of a job = number of points required to apply for it
  - ignores a lot of interesting strategic issues
  - but captures the fact that point levels reflect tensions in demand (do you like Brittany?)

- How does the price mechanism behave in presence of externalities in couple preferences?

Modelling the price mechanism

- Critical issues: joint versus individual job applications (preferences)? Transferability of points among partners?

- $I = \{1, ..., i, ...n\}$ = set of individuals
  $J = \{1, ..., j, ...n\}$ = set of jobs

- Exogenous partition $\mathcal{C}$ of I into couples $C = \{i, i'\}$

- job $j \longrightarrow$ price p_j

- allocation = bijection $\sigma : I \longrightarrow J$
  initial allocation σ^0

- $i \longrightarrow$ weak order R_i over allocations

- TWO TYPES OF BUDGET CONSTRAINT
  $(\sigma^0, p, C) \rightarrow$ **Strong budget set** $B_C^S(\sigma^0, p)$
  $= \{\sigma : p_{\sigma(i)} \leq p_{\sigma^0(i)}$ and $p_{\sigma(i')} \leq p_{\sigma^0(i')}\}$
  Weak budget set $B_i^W(\sigma^0, p)$
  $= \{\sigma : p_{\sigma(i)} + p_{\sigma(i')} \leq p_{\sigma^0(i)} + p_{\sigma^0(i')}\}$

- THREE TYPES OF EQUILIBRIUM (σ, p)
  - **Selfish** $\mathcal{E}^{self}$: $\sigma \in \arg\max_{B_i(p)} R_i$ for all i
  - **Cooperative** $\mathcal{E}^{coop}$: σ is Pareto efficient in $B_C(\sigma^0, p)$ for all C
  - **Coordinated** $\mathcal{E}^{coor}$: σ implements a Nash equ. in $B_C(\sigma^0, p)$ for all C

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	selfish	cooperative	coordinated
strong	?	?	?
weak	?	?	?

Preference domains

- **Coalition-Responsiveness** CR: each partner i has a ranking p_i of jobs, and any unilateral improvement of one partner' s situation is beneficial for the other $jp_ij' \Rightarrow (j,k) \succ_i (j',k)$ and $(j,k) \succ_{i'} (j',k)$

- **Weak lexicographic property** WL**:** facing any subset of available jobs $H \subseteq J$, each partner has a unique job in H that she wants the couple to be endowed with. Who gets that job depends on what the other partner receives
  $H = \{1, 2, 3, 4\}$, $(\mathbf{1}, \mathbf{4}) \succ_i (\mathbf{4}, 2) \succ_i (3, \mathbf{4}) \succ_i$ ...
  and $(\mathbf{3}, 4) \succ_{i'} (\mathbf{1}, \mathbf{3}) \succ_{i'} (\mathbf{3}, 2) \succ_{i'}$ ...

MAIN RESULTS

- *Set-comparison of equilibria*

strong, weak	coop	coord
self	$\subseteq$ ($\subsetneq$, WL,CR)	$\subseteq$ ($\subsetneq$, WL)
coop		$\subsetneq$ if CR $\not\subseteq$ if WL

strong self	$\not\subseteq, \not\supseteq$ (CR)	weak self
strong coop	$\not\subseteq, \not\supseteq$ (CR)	weak coop
strong coord	$\subseteq$ ($\subsetneq$, WL,CR)	weak coord

- *Properties of equilibria: individual rationality, Pareto optimality*

	IR	PO
self	yes	strong: no weak: yes if strict pref
coop	no yes if joint pref	strong: no weak: yes if strict pref
coord	no	no

- *Properties of equilibria: Core stability*

	Core	Coal Core	Cons Core
self	strong: no weak: yes	strong: no weak: yes	strong: no weak: yes
coop	strong: no weak: no	strong: no weak: yes	strong: no weak: no
coord	strong: no weak: no	strong: no weak: no	strong: no weak: no

- *Existence of strong equilibria*

- If preferences are not responsive, a strong selfish equilibrium may not exist

- The domain of responsive preferences is maximal for the existence of strong selfish equilibria

$\Rightarrow$ strong selfish, cooperative and coordinated equilibria always exist under responsiveness

key argument: standard TTC works, but equilibria are no longer necessarily outcomes of TTC. And TTC is not strategy-proof

- *Existence of weak equilibria*

- If preferences are responsive, a weak selfish equilibrium may not exist

- Same if preferences are identical

- The domain of weakly lexicographic preferences is maximal for the existence of weak cooperative equilibria

- If preferences are weakly lexicographic and responsive, a weak coordinated equilibrium always exists

key argument: a new algorithm ... not strategy-proof

CONCLUSION

- very preliminary steps towards a proper treatment of externalities in reassignment procedures

- shows the wide range and equilibrium and stability concepts, and the critical role played by preferences (and initial conditions) for existence

- many pending issues, e.g.
  - existence of strategy-proof Core stable mechanisms for job mobility
  - characterization and non-cooperative foundations of equilibrium allocations

- Doğan and Lainé (JME 2011), Aslan and Lainé (2017, SED Conference, Lund), Aslan and Lainé (2018, SCWE Conference, Séoul),