Collective Household Welfare, Intra-household Inequality and Welfare Comparisons

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Organization

- Collective Household Welfare and Intrahousehold Inequality
  - Theory
  - Application
- Welfare Comparisons across Individuals: Interpersonal Inter and Intra-household Comparisons
- Conclusions

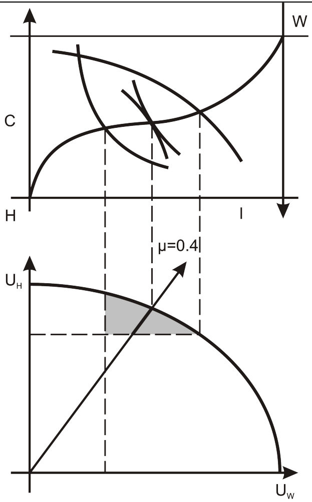
Motivation

- "The knowledge of individual well-being entails an articulate family welfare function relating the collection of potentially unequal levels of well-being of family members to an aggregate measure for the family as a whole" (Sen).
- "This *mini social choice problem*, as Sen terms it, involves "understanding" the linkages between individual behavior, household demand and the aggregation of unequal well-beings."
  - We try to "understand" these linkages.
- Chiappori and Meghir (2015) assert that the sharing rule contains all the information required to measure intra-household inequality as a general function of individual incomes.
  - We characterize such functions and estimate.

- *Theoretical Contribution*
- We show that income shares are equal to the product of two weights:
  - the Pareto weight and
  - a distribution weight reflecting income effects across individuals
- We describe the weights' role within a measure of income dispersion among household members
  - so we can investigate how, say, a tax reform impacts husbands or wives, adults or children and affects inequality
- *Empirical Contribution*
- A collective complete demand system recovering individual and household welfare and intrahousehold inequality.

The Household Mini Social Choice Problem: Graph

$$V_k = \max_{\mathbf{x}_k} \left\{ u_k(x_k) : \mathbf{p} x_k = \phi_k \right\} \qquad V = \max_{\phi_i} \left\{ U \left(V_1(\mathbf{p}, \phi_1), .., V_k(\mathbf{p}, \phi_k) \right) : \sum_k \phi_k = y \right\}$$



An Interesting Result

- From the FOC of the Lagrangian associated with the centralized program

$$\partial V(\mathbf{p}, y)/\partial y = [\partial U(u_1, \ldots, u_K)/\partial u_k][\partial V_k(\mathbf{p}_k, \phi_k)/\partial \phi_k],$$

  where each household member contributes to household utility in relation to a proportionality factor scaling the marginal utility of individual income.

- The proportionality factor can be interpreted as a weight $\mu(p, y) \in (0, 1)$ depending on both prices and income

$$\mu(p, y) = [\partial U(u_1, \ldots, u_K)/\partial u_k] = \frac{\lambda(\mathbf{p}, y)}{\lambda_k(\mathbf{p}_k, \phi_k)}.$$

Income Share, Welfare and Distribution Weights

Rewriting $\partial V(\mathbf{p}, y)/\partial y$ in elasticity form:

$$[\partial V(\mathbf{p}, y)/\partial \ln(y)][1/y] = [\partial U(\cdot)/\partial u_k]\,[\partial V_k(\mathbf{p}_k, \phi_k)/\partial \ln(\phi_k)]\,[1/\phi_k]$$

we obtain the *income sharing rule* for the k-th individual

$$\frac{\phi_k}{y} = \frac{\partial U(u_1, \ldots, u_K)}{\partial u_k}\, w_k$$

where

$$w_k = \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k)}\right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y)}\right)$$

is a *distribution weight* capturing marginal effects of income distribution.

For a Bergsonian representation $U(\cdot) = \sum_{k=1}^{K} \mu_k(\mathbf{p}, y) u_k(\mathbf{x}_k)$, then

$$\frac{\phi_k}{y} = \mu_k(\mathbf{p}, y)\, w_k,$$

and μ_k is the *welfare weight*.

The general collective identity

$$\mathbf{x}_k^C(\mathbf{p}, y, \mu(p, y)) = \mathbf{x}_k^D(\mathbf{p}_k, \phi_k(\mathbf{p}, y))$$

now specializes to

$$\mathbf{x}_k^C(\mathbf{p}, y(\mu_k(p, y)\, w_k)) = \mathbf{x}_k^D(\mathbf{p}_k, \phi_k(\mathbf{p}, y))$$

A Cobb-Douglas Example

Example

From the necessary conditions, the centralized and decentralized demands are

$$\begin{cases} x_{1i}^C = \alpha_i \mu_1 y / p_{1i}, \\ x_{2i}^C = \beta_i \mu_2 y / p_{2i}, \\ \lambda = 1/y, \end{cases} \quad \text{and} \quad \begin{cases} x_{1i}^D = \alpha_i \phi_1 / p_{1i}, \\ x_{2i}^D = \beta_i \phi_2 / p_{2i}, \\ \lambda_k = 1/\phi_k. \end{cases} \tag{1}$$

Because $x_{ki}^C = x_{ki}^D$, then $\phi_k = \mu_k y$. In the log-homothetic case, the distribution weights $w_k = 1$

$$w_k = (\lambda_k / \lambda)(\phi_k / y) = (1/\mu_k)(\mu_k) = 1$$

and intrahousehold distribution effects dissipate

$$\mathbf{x}_k^C(\mathbf{p}, y\,\mu\,(p, y)) = \mathbf{x}_k^D(\mathbf{p}_k, \phi_k(\mathbf{p}, y)).$$

General Representation of Individual and Household Welfare

- Welfare function for the k-th individual

$$V_k(\mathbf{p}_k, \phi_k) = f_k\left(\frac{g_k(\phi_k, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)g_k(\phi_k, \mathbf{p}_k)}\right),$$

where f_k is an increasing function, $B_k(\mathbf{p}_k)$, $C_k(\mathbf{p}_k)$ and $g_k(\cdot)$ are such that $V_k(\cdot)$ is homog. of degree zero in $(\phi_k, \mathbf{p}_k)$.

  - This flexible utility specification includes many models as special cases such as QES, QAIDS.

- Household welfare aggregates as

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k V_k(\cdot) = \sum_{k=1}^{K} \mu_k f_k\left(\frac{g_k(\mu_k w_k y, \mathbf{p}_k)}{B_k(\mathbf{p}_k) + C_k(\mathbf{p}_k)g_k(\mu_k w_k y, \mathbf{p}_k)}\right)$$

Intra-household Inequality

- This general collective household welfare function can be disaggregated into
  - an efficiency component as in a unitary framework with equal distribution
  - an equity component describing the dispersion in individual prices and resources.
- For example, the QES specification is a monotonic function of household income y deflated by household price index P_0

$$V(\mathbf{p},y) = \ln(y) - \ln P_0(\mathbf{p},y,\boldsymbol{\mu}) = \ln(y) - \left(\sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k)) + \sum_{k=1}^{K} \mu_k \ln(\mu_k) \right).$$

The equity component (log of price index P_0) is composed of:

  - the weighted sum $\sum_{k=1}^{K} \mu_k \ln(B_k(\mathbf{p}_k))$ describing dispersion in individual prices
  - the Shannon entropy index $\sum_{k=1}^{K} \mu_k \ln(\mu_k)$ capturing inequality in the distribution of resources.

Application: Estimation of μ_k, w_k and Intra-household Inequality

Consider the AIDS indirect utilities

$$V_k(\mathbf{p}_k, \phi_k) = \frac{\ln(\phi_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)}.$$

The associated household welfare function is

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) + \ln(\mu_k w_k) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)},$$

and the intra-household income share is

$$\frac{\phi_k}{y} = \mu_k w_k,$$

where μ_k is the welfare weight and $w_k = \left(\frac{\partial V_k(\mathbf{p}_k, \phi_k)}{\partial \ln(\phi_k)}\right) / \left(\frac{\partial V(\mathbf{p}, y)}{\partial \ln(y)}\right)$ is the distribution weight.

The Distribution Weight

The distribution weight is

$$w_k = \left(\frac{1}{B_k(\mathbf{p}_k)}\right) \Big/ \left(\sum_{k=1}^{K} \frac{\mu_k}{B_k(\mathbf{p}_k)}\right).$$

- In the case of two agents $k = 1, 2$, the ratio of the distribution weights does not depend on the welfare weights $\frac{w_1}{w_2} = \frac{B_2}{B_1}$.
- Similarly, the ratio of the welfare weights is independent of the distribution weights $\frac{\mu_1}{\mu_2} = \frac{\phi_1 B_1}{\phi_2 B_2}$.

The Welfare Weight

The welfare weights are

$$\mu_1 = \frac{\phi_1 B_1}{\phi_1 B_1 + \phi_2 B_2}, \text{ and } \mu_2 = \frac{\phi_2 B_2}{\phi_1 B_1 + \phi_2 B_2}.$$

Substituting these expressions in the condition $\sum_{k=1}^{K} \mu_k w_k = 1$ and using the relationship that $w_2 = w_1 \frac{B_1}{B_2}$, we obtain the distribution weights

$$w_1 = \frac{\phi_1 B_1 + \phi_2 B_2}{B_1 y}, \text{ and } w_2 = \frac{\phi_1 B_1 + \phi_2 B_2}{B_2 y}.$$

Intra-household Inequality

- Given the knowledge of the welfare and distribution weights, we compute the measure of intrahousehold inequality.
- Rewriting the household welfare function as

$$V(\mathbf{p}, y) = \sum_{k=1}^{K} \mu_k \frac{\ln(y) - \ln(A_k(\mathbf{p}_k))}{B_k(\mathbf{p}_k)} + \sum_{k=1}^{K} \frac{1}{B_k(\mathbf{p}_k)} (\mu_k \ln(\mu_k w_k)),$$

we observe that

- the first term of the household welfare function is the deflated level of household income
- the second term is a measure of dispersion in individual welfare levels similar to a Theil inequality index scaled by the inverse of the price index $B_k(\mathbf{p}_k)$.

Results: Collective Demand

- A collective AIDS system of budget shares linear in the log of total expenditure
- The application uses Italian household budget data for the year 2007 about couples without children
- We run the STATA command as
    - cquaids *wfood whouse wcloth wtransp wothers*, lnpr(*lnpfood lnphouse lnpcloth lnptransp lnpothers*) lnexp(*lnx*) demo(*r1 r2 r3 r4 tj ts winter landlord nauto*) d_f(*ageratio eduratio ratiop*) err(*error_lnx*) ngroup(*nfemale nmale*) wgt(*rs_female1 rs_male1*) full model(*linear*)

- A demographically modified collective share equation ω_i (??) becomes

$$\omega_i = \alpha_i + t_i(\mathbf{d}) + \sum_j \gamma_{ji} \ln p_j + \sum_{k=1}^{2} \beta_{ki} \left(\ln \phi_k^* - \ln A_k(\mathbf{p}_k) \right) + \varepsilon_i, \tag{2}$$

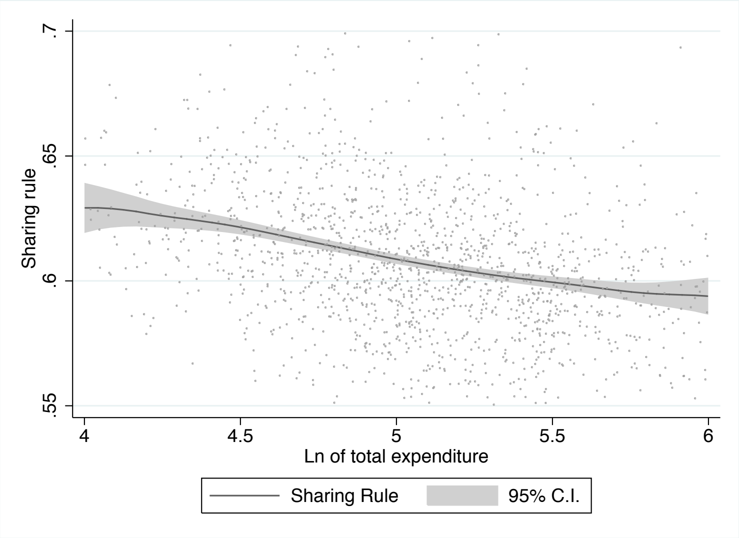
with

$$\phi_k = m_k(\mathbf{z}) y_k, m_k(\cdot) \in \left(0, \frac{y}{y_k}\right).$$

- Pricing private equivalents at the Lindhal (only in the public goods case) shadow prices $A'p$ gives the resource share $\mu(p, y)$.
- Each household member, in a decentralized fashion, chooses x_k as the bundle maximizing $u_k(\mathbf{x}_k)$ subject to the shadow budget constraint

$$\sum_k A_k p^k x^k = \mu y$$

The Income Share $\frac{\phi_x}{y}$ (male)



The Welfare μ and Distribution w Weights

Table: Welfare Levels, Pareto and Distribution Weights

	Mean	Std. Dev.	Min	Max
Individual welfare levels				
Female	5.100	0.662	3.110	8.365
Male	2.888	0.396	1.843	4.271
Household welfare levels	3.688	0.466	2.566	5.281
Pareto weight				
Female	0.363	0.016	0.307	0.424
Male	0.637	0.016	0.576	0.693
Distribution weight				
Female	1.374	0.065	1.175	1.644
Male	0.789	0.023	0.715	0.872

Intra-Household Inequality

Table: Intra-Household Inequality

	Mean	Std. Dev.	Min	Max
All sample	−0.336	0.028	−0.434	−0.244
Poor	−0.319	0.023	−0.382	−0.265
Rich	−0.365	0.024	−0.434	−0.284

- This index allows comparisons of two households with similar levels of household income, but differing for the intra-household distribution of resources.
- The distribution of resources within less affluent Italian households is more equitable as compared to the distribution of resources in rich families.

Inter-Personal Welfare Comparisons

Definition

Interpersonal Scales. $S^0_{kj}(p, v, d_k, d_j)$ is the income needed to allow individual j to reach the same utility level of individual k,

$$V_j(p, \phi_k/S^0_{kj}, d_j) = V_k(p, \phi_k, d_k)$$

or assuming equivalence scale exacteness or independence of the level of utility at which comparisons are based

$$S^0_{kj}(p, v, d_k, d_j) = \frac{C_k(p, v_k, d_k)}{C_j(p, v_j, d_j)} = \frac{C_k(p, d_k)C(v, p)}{C_j(p, d_j)C(v, p)} = S^0_{kj}(p, d_k, d_j).$$

- When we are comparing the same individual $k = j$ in different situations, we obtain an *indifference scale*.

Inter-Personal Welfare Comparisons 2

- In light of our results we can decompose the *interpersonal scales* as

$$S_{kj} = \frac{\phi_k}{\phi_j} = \frac{\mu_k^0 w_k^0}{\mu_j^1 w_j^1} \frac{y^0}{y^1}.$$

- We now are in the position to address some open issues as exemplified by Pendakur (2018): "after converting an able-bodied person and disabled person into as-if singles, we must still account for the fact that the disabled person has greater needs than does the able person. Thus, indifference scales and equivalence scales are complements, not substitutes."

Intra and Inter Household Comparisons across Individuals

Situation	Intra (household)	Inter (household)	
Intra (person)	1) $IS_k^{01} = \frac{\mu_k^0 w_k^0}{\mu_k^1 w_k^1}$	2) $\bar{IS}_k^{01} = \frac{\mu_k^0}{\tilde{\mu}_k^1} \frac{1}{\tilde{w}_k^1} \frac{y^0}{\tilde{y}^1}$	IS
Inter (person)	4) $ES_{kj}^0 = \frac{\mu_k^0 w_k^0}{\mu_j^0 w_j^0}$	3) $\bar{ES}_{kj}^0 = \frac{\mu_k^0 w_k^0}{\tilde{\mu}_j^0 \tilde{w}_j^0} \frac{y^0}{\tilde{y}^0}$	ES

1. *intra personal-intra household*: the same person before and after an injury staying in the same hh
2. *intra personal-inter household*: same person before and after marriage or divorce
3. *inter personal-intra household*: brother and sister or husband and wife
4. *inter personal-inter household*: two individuals in different households

Conclusions

- We recover the relationships between centralized and decentralized programs to investigate the relationship between sharing rule, hh welfare and intra-hh inequality.
- We analyze the properties of the sharing rule as a function of two weights:
    - the Bergsonian welfare weights μ
    - distribution weights reflecting income effects across household members w.
- Intra-household inequality is described by a family of entropy indexes.
- The empirical application estimates μ and w: richer couples distribute resources among their members less equally.
- We discuss implications for the implementation of welfare comparisons across individuals in different situations: soon the estimates.